The Standard Model and Feynman Diagrams

By: Michael Kolacki

Introduction

- Elementary Particle– something with no known substructure
- Types: Leptons, Quarks, and Bosons
- **Leptons:**
  - Fermions (spin ½)
  - 3 different families (e, μ, τ) each containing a particle and neutrino pair (e.g.: e and ν_e)
  - The base particles of each family have charge $-e$ while the neutrinos have zero charge
  - All six particles also have an associated anti-particle with identical mass and opposite charge
  - Cannot interact via the strong force because they do not carry color (more on this later)
  - Quantum number L
    - Three types, L_e, L_μ, and L_τ; one for each family. Particles in a particular family carry a respective L-value of 1, with their anti-particles carrying a value of -1. All other particles have L values of 0.
    - Family specific L values are generally conserved, however this conservation may be violated by neutrino oscillation
    - Overall L-value ($L = L_e + L_\mu + L_\tau$) is always conserved, except perhaps at the conception of the universe.

Introduction

- **Quarks:**
  - Fractionally charged particles which can't exist independently
  - Also fermions, with six different types (flavors), each with an associated antiparticle
  - Several different quantum numbers– Baryon Number, S, C, B, T, L, and Color

Type of Quark	Symbol	Charge	Quantum Numbers				
			Baryon Number	Charm (C)	Strangeness (S)	Topness (T)	Bottomness (B)
Up	u	2e/3	1/3	0	0	0	0
Down	d	-e/3	1/3	0	0	0	0
Charmed	c	2e/3	1/3	1	0	0	0
Strange	s	-e/3	1/3	0	-1	0	0
Top	t	2e/3	1/3	0	0	1	0
Bottom	b	-e/3	1/3	0	0	0	-1

  - Color is variable and changes with strong interaction between "charges" of Red (r), Green (g), and Blue (b)
  - Antiparticles have identical mass, but negative values for charge, Baryon Number, S, C, B, and T
  - These six quarks and their anti-particles constitute all of the Hadrons.

Introduction

- **Hadrons:**
  - Particles that interact via the strong force
  - All have integral charge and Baryon Number
  - 2 sub-classes: Mesons and Baryons
  - Baryons: Comprised of 3 quarks with a Baryon Number of 1
    - Baryon Number is always conserved, except again at the beginning of the universe
    - Contains the nucleons
    - Have half-integral spins (fermions)
  - Mesons: Comprised of a quark - anti-quark pair, providing a Baryon Number of 0 and spin 0
    - Because they have integral spin, they are also bosons
  - Note: Strangeness and charm are conserved in strong and electromagnetic interactions, but not weak interactions

    Charge, linear momentum, and angular momentum are always conserved

Introduction

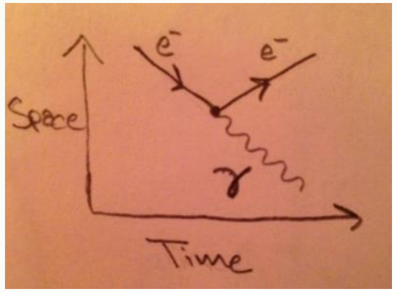
- **Bosons:**
  - Carriers/mediators of force with integer spin
  - 6 elementary types, with 4 Gauge Bosons

Boson		Mass	Spin	Charge	Color	Force Carried/Mediated
Gauge	Photon (γ)	0	1	0	0	Carrier of electromagnetism
	W^+, W^-	80.40 GeV/c^2	1	1, -1	0	Mediators of some weak interactions
	Z	91.19 GeV/c^2	1	0	0	Mediator of some weak interactions
	Gluon	0	1	0	1 of 8 color – anti-color combinations	Carrier of the strong force
Others	Higgs (H^0)	>114GeV/c^2	0	0	0	Gives mass to W and Z bosons, quarks and massive leptons
	Graviton	0	2	0	0	Mediator of the gravitational force

- Note:
  - W carries a charge, and gluons carry a color – anti-color pair (1 of 8)
  - Higgs proposed as a possible explanation of why the $W\pm$ and the Z have masses while the photon has no mass.
    - Higgs field: permeates space and gives mass to particles based on level of interaction

The Standard Model and Feynman Diagrams

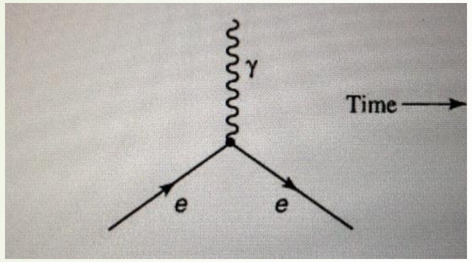
- Combinatorial evaluation of Quantum Electrodynamics and Weak Interaction (Electroweak interaction) and Quantum Chromodynamics, but not of gravity
- Interactions are most easily represented with Feynman Diagrams, which have the following general structure:



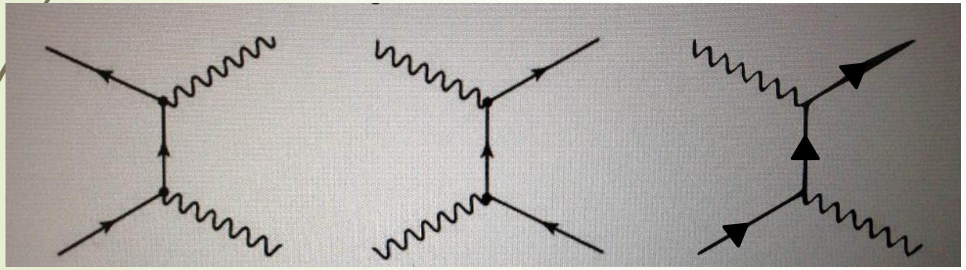
- Each diagram stands for a number which corresponds to the amplitude of a particular interaction. The sum total of all diagrams represents the actual process.
- Diagrams consist of both "internal" lines (representing virtual particles), "external" lines (real/observable particles), and vertices
- Diagram contributes less and less with an increased count of vertices
- Diagrams do not represent particle trajectories.
- Conservation laws must be enforced within the diagram, otherwise it is not valid.

The Standard Model and Feynman Diagrams

- **QED:**
  - May involve any charged particle (no neutrinos) and a photon
  - Primitive Vertex:



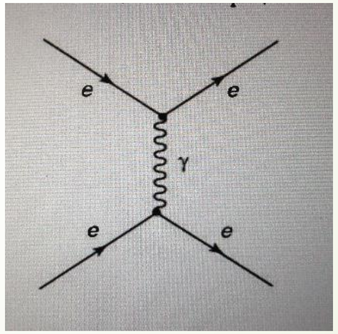
  - Charge is conserved in the process of the interaction
  - Notable Diagrams:



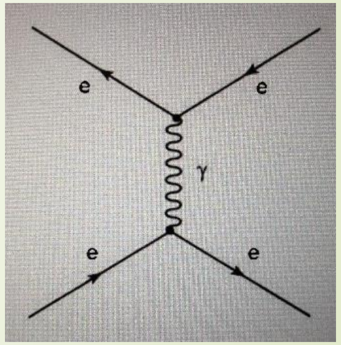
Pair Annihilation Pair Production Compton Scattering

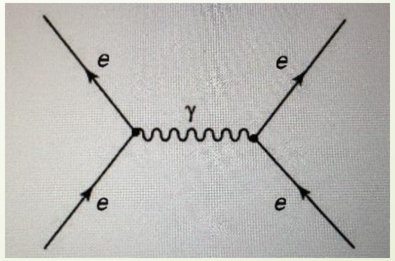
  - Note: Diagrams are allowed to be "twisted" (laid on their sides)

MØller Scattering



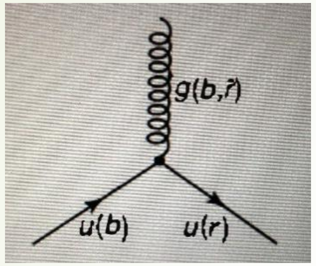
Bhabha Scattering

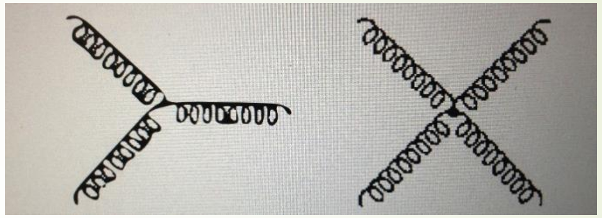




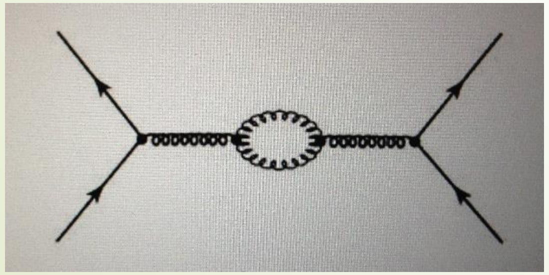
The Standard Model and Feynman Diagrams

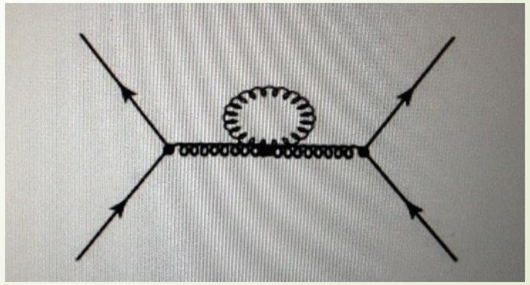
- **QCD:**
  - Fundamental Process:





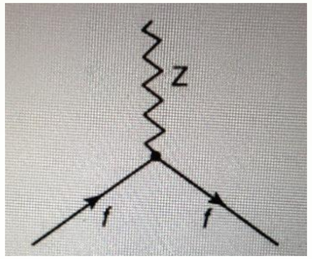
  - Because gluons carry a color charge they are capable of interacting with one another, leading to 2 more primitive vertices
  - Color is conserved, just as charge is for QED
  - Color changes between quarks during strong interaction, but flavor does not
  - The coupling of gluons with other gluons makes things like loops possible in Feynman diagrams for strong interaction



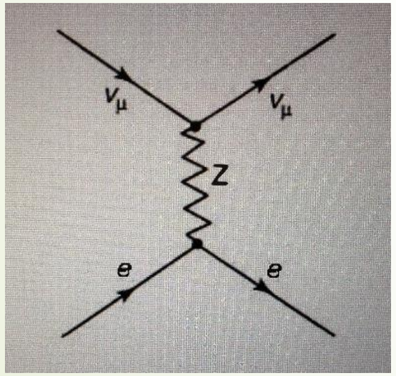


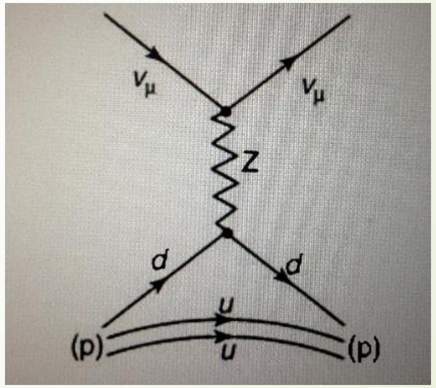
The Standard Model and Feynman Diagrams

- **Weak Interaction:**
  - 2 Kinds: Charged – mediated by $W\pm$, and neutral interactions by the Z
  - Fundamental vertex for neutral interaction (the Z):



  - Most commonly associated with neutrino scattering
  - Ex.

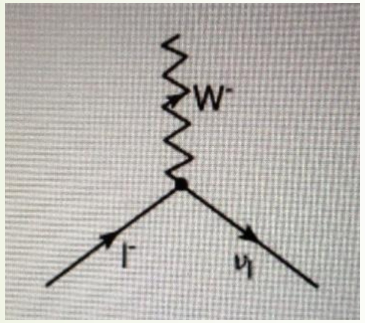


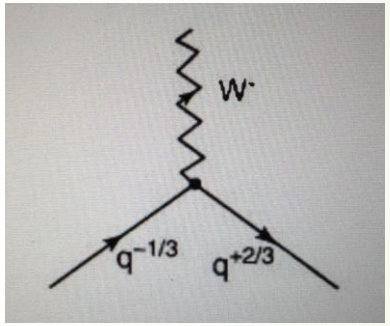


  - So far, all primitive vertices have had the same particles leaving as having entered, except for color having changed in strong interaction, though flavor was still preserved. This is not the case with the $W\pm$

The Standard Model and Feynman Diagrams

- **Weak Interaction (cont.):**
  - For Leptons: A lepton converts to its corresponding neutrino with emission (W^-) and vice versa with absorption (W^+)
  - For quarks: Color is preserved while flavor changes, with q_1 and q_2 generally being in the same generation.
  - Fundamental vertex (for leptons then quarks respectively):

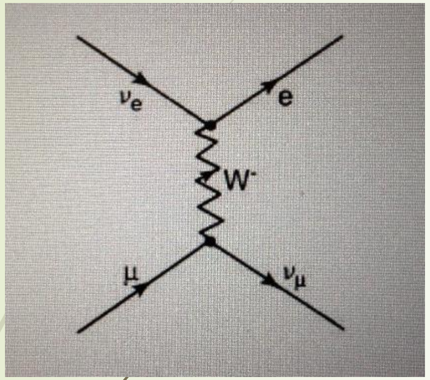


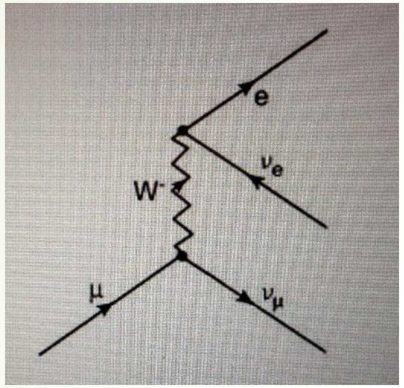


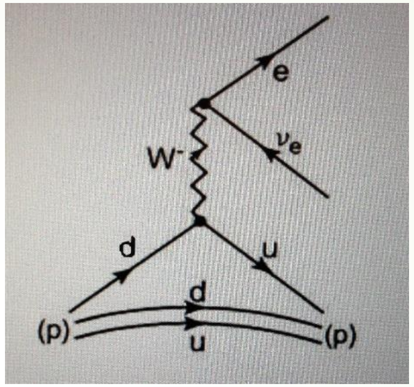
  - One would assume that there would be no cross generational transitions, as was the case for leptons, conserving upness – plus – downness, however the reaction is not this simple. In reality, a u, c, or t quark is coupled to d', s' and b' respectively, where the latter are derived according to the following matrix operation
  - $$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} * \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$ where the second matrix has experimental values of $$\begin{bmatrix} .974 & .227 & .004 \\ .227 & .973 & .042 \\ .008 & .042 & .999 \end{bmatrix}$$
  - This allows for an enormous variety of processes

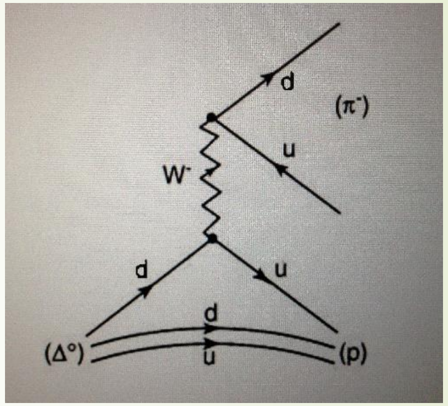
The Standard Model and Feynman Diagrams

- **Weak Interaction (cont.):**
  - Examples of W^- interaction:

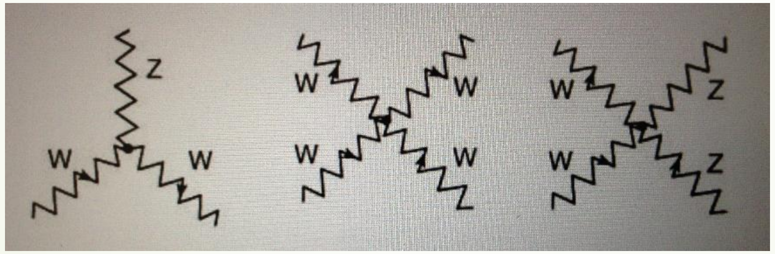




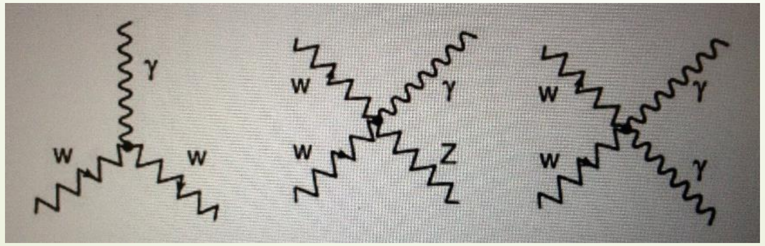




  - The W and Z are also capable of being coupled together in the following manners:



  - And finally, because the W is charged, it is also capable of interacting/coupling with photons



Feynman Diagrams – Process of Calculation

1) Label 4 momenta for all incoming and outgoing lines $p_1, p_2, \dots p_n$ where p_i is forward with time. Also label 4-momenta for all internal lines as $q_1, q_2, \dots q_n$ where the direction of q_i may be chosen.

2) Write for each vertex a factor of $-ig_f$ where g_f is the coupling constant representing the strength of interaction between lines involved with the vertex for the particular type of force f and is represented by either

$$g_e = \sqrt{4\pi\alpha}\ , \qquad g_w = \frac{g_e}{\sin\theta_w}\ , \qquad g_z = \frac{g_e}{\sin\theta_w \cos\theta_w}\ , \qquad \text{or} \quad g_s = \sqrt{4\pi\alpha_s} = 1.214$$

where $\theta_w = 28.75°$ and $\alpha = \frac{e^2}{\hbar c}$

3) For each internal line, write another factor:

spin 0: $\frac{i}{q_n^2 - m_n^2 c^2}$ spin ½: $\frac{i m_n c}{q_n^2 - m_n^2 c^2}$ spin 1: (massless) $-\frac{i g_f}{q_n^2}$ or (massive) $-\frac{i\left[g_f - \frac{q_1 q_2}{m_n c^2}\right]}{q_n^2 - m_n^2 c^2}$

These factors are called propagators.

Feynman Diagrams – Process of Calculation

4) Also write for each vertex a delta function

$$(2\pi)^4\delta^4(m_1 + m_2 + m_3)$$

where m_i is the momenta of 1 of 3 respective lines connected to the vertex. If m_i enters the vertex it is positive. If it exits the vertex it is negative. This function ensures that if momentum is not conserved, the whole function goes to 0.

5) Now, for each internal line write a final factor of $\frac{1}{(2\pi)^4}d^4q_n$ and integrate over all internal momenta.

6) The result should include the delta function factor of $(2\pi)^4\delta^4(p_1 + p_2 + \cdots p_n)$ where p_i may be negative. This result verifies an overall conservation of momentum and energy. To achieve a final function, this factor may be erased and the resulting function multiplied by i. This resultant function is $\mathcal{M}$, amplitude.

References

- Thornton, S. T., & Rex, A. (2013). Modern Physics for Scientist and Engineers (4th ed.) Boston, Mass.: Brooks/Cole.
- Griffiths, David (2008). Introduction to Elementary Particles (2nd ed., revised). Portland, OR.: Wiley-VCH
- The Particle Adventure, "The Standard Model," www.particleadventure.org/standard-model.html, accessed September 23, 2015.